

# Phenomenology and Cosmology of Supersymmetric Grand Unified Theories

Constantinos Pallis

A Dissertation in Physics  
Supervisor: Prof. George Lazarides

*Physics Division  
School of Technology  
Aristotle University of Thessaloniki  
Thessaloniki, GR 540 06, Greece*

**June 2000**

# Summary

The Minimal Supersymmetric Standard Model is the most promising extension of the Standard Model. On the other hand, the Standard Big Bag Cosmology combined with inflation provides a consistent picture for the universe evolution. The combination of these two theories can successfully address the Cold Dark Matter problem. Indeed, the Lightest Supersymmetric Particle is the most plausible Cold Dark Matter candidate.

In this thesis, we calculate the cosmological relic density of a Bino-like Lightest Supersymmetric Particle in the framework of the Minimal Supersymmetric Standard Model. We include annihilation and coannihilation effects of the Bino with the lightest stau and other sleptons, which happen to have comparable masses with it. Coannihilation turns out to be of crucial importance for reducing the Bino relic density to an acceptable level. Requiring the Bino relic density to be in the cosmologically allowed region, derived from the mixed or the pure (in the presence of a nonzero cosmological constant) Cold Dark Matter scenarios for large scale structure formation in the universe, one can restrict the relative mass splitting between the Lightest and the Next-to-Lightest Supersymmetric Particle. Phenomenological constraints, also, result from the inclusion of the supersymmetric corrections to the CP-even Higgs boson and  $b$ -quark masses and the branching ratio of  $b \rightarrow s\gamma$ . We impose these constraints on the parameter space of two versions of the Minimal Supersymmetric Standard Model, employing radiative electroweak breaking with universal boundary conditions and gauge coupling unification.

In the first version of the model we assume Yukawa coupling unification and boundary conditions from gravity-mediated supersymmetry breaking. For  $\mu < 0$ , the branching ratio of  $b \rightarrow s\gamma$  is compatible with data but the  $b$ -quark mass after including supersymmetric corrections exceeds the experimental limits. The Bino mass can range between 215 GeV and 770 GeV with the lightest stau mass being 8-0% larger. For  $\mu > 0$ , the predicted  $b$ -quark mass is experimentally acceptable and there is a sizable fraction of the parameter space allowed by  $b \rightarrow s\gamma$ , where Bino relic density is below the upper bound from Cold Dark Matter considerations.

In the second version of the model we assume boundary conditions from the Hořava-Witten Theory and restrict the parameter space by simultaneously imposing the phenomenological and cosmological constraints. Complete and  $t - b$  Yukawa unification can be excluded. Also,  $b - \tau$  Yukawa unification is not so favored since it, generally, requires almost degenerate lightest and next-to-lightest sparticle masses. The no Yukawa unification case is the most natural one since it can avoid this degeneracy. The lightest sparticle mass can range between 70 GeV and 670 GeV with the lightest stau mass being 93-0% larger.<sup>1</sup>

## Publications

- [1] M. E. Gómez, G. Lazarides and C. Pallis, *Supersymmetric Cold Dark Matter with Yukawa Unification*, Phys. Rev. **D61**, 123512 (2000), [hep-ph/9907261](#).
- [2] M. E. Gómez, G. Lazarides and C. Pallis, *Yukawa Unification,  $b \rightarrow s\gamma$  and Bino-Stau Coannihilation*, [hep-ph/0004028](#) (to appear in Phys. Lett. B).
- [3] S. Khalil, G. Lazarides and C. Pallis, *Cold Dark Matter and  $b \rightarrow s\gamma$  in the Hořava-Witten Theory*, [hep-ph/0005021](#).

---

<sup>1</sup>The full postscript version of the dissertation (135+7 pages in Greek) is available from <http://users.auth.gr/~kpallis/Thesis.ps>

# Contents

<b>1 Introduction</b>	<b>1</b>
<b>2 Minimal Supersymmetric Standard Model</b>	<b>3</b>
2.1 Introduction .....	3
2.2 Supersymmetric Gauge Theories .....	3
2.3 The MSSM structure .....	9
2.4 Higgs Mechanism in the MSSM .....	16
2.5 The MSSM Sparticle Spectrum .....	20
2.6 The Fermion-Sfermion-Bino vertex .....	24
<b>3 Phenomenological constraints-Corrections</b>	<b>25</b>
3.1 Introduction .....	25
3.2 Corrections to the Higgs boson masses .....	25
3.3 Corrections to the fermion masses .....	27
3.4 The branching ratio of $b \rightarrow s\gamma$ .....	30
<b>4 Standard Cosmology</b>	<b>35</b>
4.1 Introduction .....	35
4.2 Cosmological Principle .....	35
4.3 Cosmological Dynamics .....	38
4.4 Cosmological Thermodynamics .....	40
4.5 The Hot Big Bang .....	44
4.6 Inflation .....	49
4.6 Critics of the Cosmological Model .....	53
<b>5 Supersymmetric Dark Matter</b>	<b>54</b>
5.1 Introduction .....	54
5.2 Dark Matter scenarios .....	54
5.3 LSP Relic Density .....	59
5.4 Calculation method of $\langle \sigma_{ij} v_{ij} \rangle$ .....	63
5.5 Application to a Bino-like LSP .....	67

<b>6 MSSM with Yukawa unification</b>	<b>76</b>
6.1 Introduction .....	76
6.2 SUSY GUTs with Yukawa unification .....	76
6.3 Gauge and Yukawa unification .....	77
6.4 Parametric analysis .....	79
6.5 Phenomenological considerations .....	84
6.6 Cosmological results .....	86
6.7 Conclusions .....	91
6.8 Beyond Yukawa unification .....	92
<b>7 MSSM and the Hořava-Witten Theory</b>	<b>97</b>
7.1 Introduction .....	97
7.2 The Hořava-Witten Model .....	97
7.3 Numerical analysis .....	99
7.4 Parametric analysis .....	99
7.5 Phenomenological considerations .....	102
7.6 Cosmological results .....	103
7.7 Conclusions .....	109
<b>8 Conclusions</b>	<b>110</b>
<b>Bibliography</b>	<b>112</b>
<b>A' Renormalization Group Equations</b>	<b>116</b>
A'.1 Introduction .....	116
A'.2 RGEs for Gauge and Yukawa couplings .....	116
A'.3 RGEs for Soft Breaking Terms .....	118
<b>B' Feynman rules and applications</b>	<b>120</b>
B'.1 Introduction .....	120
B'.2 Conventions-Normalizations .....	120
B'.3 Feynman rules .....	122
B'.4 Calculation of amplitudes .....	131
<b>C' Acronyms</b>	<b>134</b>